## MATHEMATICS (MATH)

## MATH 10 Basic Mathematics and Elementary Algebra (4 crs)

Prerequisite: For students with minimal high school mathematics or appropriate score on the mathematics placement test.

- No credit if taken after credit is earned in any higher-numbered mathematics course. This course is graded on a letter grade basis. The grade received is used in calculating semester, resident, and total GPAs.

Develops basic mathematics and elementary algebraic skills necessary for further study of algebra.

Grading Basis: No Auditors
Lecture/Discussion Hours: 1
Lab/Studio Hours: 3

## MATH 20 Intermediate Algebra (4 crs)

Prerequisite: A grade of C or above in MATH 10, or one year of collegepreparatory algebra and an appropriate score on the mathematics placement test.
-This course is graded on a letter grade basis. The grade received is used in calculating semester, resident, and total GPAs. No credit if taken after credit is earned in any higher-numbered mathematics course except MATH 106 or MATH 107.

The real number system, linear equations and inequalities, operations on polynomials and rational expressions, factoring, solutions of quadratic equations, Cartesian coordinates, functions, and related applications.

Grading Basis: No Auditors
Lecture/Discussion Hours: 1
Lab/Studio Hours: 3

## MATH 50 Supplemental Math Preparation (1 cr)

Prerequisite: Suitable mathematics placement test score and concurrent enrollment with MATH 104, 108, 109, 201 or 246.
Development of foundational skills and concepts to allow concurrent enrollment in certain credit-bearing mathematics courses in lieu of prerequisite enrollment in MATH 20.

Grading Basis: A-F Grades Only
Lecture/Discussion Hours: 1
Lab/Studio Hours: 0

## MATH 104 Finite Mathematics with Applications (4 crs)

Prerequisite: Grade of C or above in MATH 20, or 2 yrs college-prep algebra + suitable math placement score, or concurrent enrollment in MATH 50 + suitable placement score, or grade of C or above in MATH 50. No credit after MATH 114, 212, 314 or 346

- May not be counted for credit toward a major or minor in mathematics.

This course will cover applications from Economics, Social and Biological Sciences. The topics include sets, functions, counting principles, probability, solutions of linear systems by graphical and algebraic methods, matrices, linear programming, problem formulation, and using graphs and trees

Attributes: GE IB Communication-Analytical Skills, LE-S2 Mathematics Lecture/Discussion Hours: 4 Lab/Studio Hours: 0

## MATH 106 Introduction to Mathematical Thinking (4 crs)

Prerequisite: MATH 20, or two years of college-preparatory algebra and a suitable mathematics placement test score. Credit may not be earned in both MATH 106 and MATH 101.

- May not be counted for credit toward a major or minor in mathematics.

An accessible introduction to some interesting mathematical topics that require analytical thinking and abstract reasoning skills.

Attributes: GE IB Communication-Analytical Skills, LE-S2 Mathematics Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

MATH 107 Mathematics in Latin American Cultures (4 crs)
Prerequisite: MATH 20, or two years of college-preparatory algebra and a suitable mathematics placement test score.

- May not be counted for credit toward a major or minor in mathematics.

This course introduces important mathematical concepts and topics, such as number and arithmetic systems, symmetry, and data structure, using the cultural lenses of pre-Columbian Latin American indigenous cultures.

Attributes: GE IB Communication-Analytical Skills, LE-S2 Mathematics Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 108 Earth Algebra (4 crs)

Prerequisite: Grade of C or above in MATH 20, or 2 yrs college-prep algebra + suitable math placement score, or concurrent enrollment in MATH 50 + suitable placement score, or grade of C or above in MATH 50.

- May not be counted for credit toward a major or minor in mathematics.

Modeling real data drawn from biology, the environment, finance, and other physical systems. Topics include the development of linear, polynomial, exponential and logarithmic functions, and the rate of change in an applied setting.

Attributes: GE IB Communication-Analytical Skills, LE-R3 Civic and Environmental Issues, LE-S2 Mathematics Lecture/Discussion Hours: 4 Lab/Studio Hours: 0

## MATH 109 Algebra for Calculus (4 crs)

Prerequisite: Grade of C or above in MATH 20, or 2 yrs college-prep algebra + suitable math placement score, or concurrent enrollment in MATH 50 + suitable placement score, or grade of C or above in MATH 50.

- May not be counted for credit toward a major or minor in mathematics.

Algebraic concepts, techniques, and applications including polynomial and rational expressions, linear and quadratic equations, complex numbers, inequalities, absolute value, functions and graphs, exponential and logarithmic functions, systems of equations and inequalities, and zeros of polynomials. This course is for students pursuing degree programs that require calculus.

Attributes: LE-S2 Mathematics
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 111 A Short Course in Calculus (4 crs)

Prerequisite: MATH 109, or a grade of B or above in MATH 108, or three years of above-average work in college-prep MATH and a suitable mathematics placement test score.

- May not be counted for credit toward a major or minor in mathematics.

Primarily for students in biology, business, economics, psychology, and sociology. Functions, limits, continuity, derivatives, integrals, and applications.

Attributes: GE IB Communication-Analytical Skills, LE-S2 Mathematics Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 112 Precalculus Mathematics (4 crs)

Prerequisite: MATH 108, or MATH 109, or three years of above-average work in college-prep MATH and a suitable mathematics placement test score.

- May not be counted for credit toward a major or minor in mathematics.

Prepares students to enter the Math 114, Math 215, Math 216 sequence. Includes absolute value; logarithmic, exponential, and trigonometric functions; inequalities; conic sections; complex numbers; and topics from theory of equations.

Attributes: LE-S2 Mathematics
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 113 Trigonometry (2 crs)

Prerequisite: MATH 109, or a grade of B or above in MATH 108, or three years of above-average work in college-prep MATH and a suitable mathematics placement test score. Credit may not be earned in both MATH 113 and MATH 112.

- May not be counted for credit toward a major or minor in mathematics.

Graphs, properties, and geometric significance of trigonometric functions of a real variable. Other topics include trigonometric equations and identities, inverse trigonometric functions and applications.

Lecture/Discussion Hours: 2
Lab/Studio Hours: 0

## MATH 114 Calculus I (4 crs)

Prerequisite: MATH 109 and MATH 113, or MATH 112, or four years of aboveaverage work in college-prep mathematics, including one semester of trigonometry and a suitable mathematics placement test score.
Limits, theory, and application of the derivative; introduction to integration.
Attributes: GE IB Communication-Analytical Skills, LE-S2 Mathematics Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 150 Introduction to the Actuarial Career (1 cr)

- No credit if take after Math 291 when offered as Introduction to the Actuarial Career

Designed to start or further efforts in understanding the actuarial profession and to prepare students for an actuarial job search.

Lecture/Discussion Hours: 0
Lab/Studio Hours: 3

## MATH 173 Introduction to Math Teaching (1 cr)

- Intent to major in Mathematics Teaching or Comprehensive Mathematics Teaching is recommended for enrollment.

This course seeks to extend student understanding of the mathematics teaching profession and address introductory topics and skills relevant to the teaching and learning of mathematics.

Seminar Hours: 1

## MATH 201 Number and Operations I (2 crs)

Prerequisite: Grade of C or above in MATH 20, or 2 yrs college-prep algebra + suitable math placement score, or concurrent enrollment in MATH 50 + suitable placement score, or grade of $C$ or above in MATH 50. ELED and SPED majors/minors only.
Mathematical knowledge for teachers based on practice/content standards.
Topics include: number systems, number theory, and integer algorithms.
Lecture/Discussion Hours: 2
Lab/Studio Hours: 0

## MATH 202 Number and Operations II (2 crs)

Prerequisite: MATH 201. Limited to elementary and special education majors and minors.
Mathematical knowledge for teachers based on practice/content standards. Topics include: fractions, ratios/proportions, and decimals.

Attributes: LE-S2 Mathematics
Lecture/Discussion Hours: 2
Lab/Studio Hours: 0

MATH 203 Mathematics for Elementary/Middle School Teachers I (3 crs)
Prerequisite: A grade of C or above in MATH 20, or two years of collegepreparatory algebra and a suitable math placement test score. Limited to elementary or special education majors and minors.

- May not be counted for credit toward a major or minor in mathematics.

An integrated study of mathematics content, methods, and curricula. Topics include problem solving, reasoning, whole numbers and numeration with an emphasis on cognitively guided instruction, elementary number theory, integers.

Attributes: GE IB Communication-Analytical Skills
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

MATH 204 Mathematics for Elementary/Middle School Teachers II (3 crs)
Prerequisite: MATH 203. Limited to elementary and special education majors and minors. Minimum sophomore standing.

- Counts only toward the elementary teaching minor in mathematics.

An integrated study of mathematics content, methods, and curricula. Topics include number theory, the rational numbers, ratio and proportion, the real number system and probability. Emphasis on communication and algebraic thinking. Practicum experience included.

Attributes: GE IB Communication-Analytical Skills
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 212 Calculus and Business Mathematics (3 crs)

Prerequisite: MATH 111 or MATH 114.

- May not be counted toward a major in mathematics.

Multivariate calculus, matrix algebra, linear programming, mathematics of finance, and business-oriented applications.

Attributes: GE IB Communication-Analytical Skills
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 215 Calculus II (4 crs)

Prerequisite: MATH 114
Applications and techniques of integration; improper integrals; sequences and series; power series and Taylor's formula.

Attributes: GE IB Communication-Analytical Skills
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 216 Calculus III (4 crs)

Prerequisite: MATH 215
Introduction to functions of several variables, including partial derivatives, multiple integrals, the calculus of vector-valued functions, and Green's Theorem, Stokes' Theorem, and the Divergence Theorem.

Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 246 Elementary Statistics (4 crs)

Prerequisite: Grade of C or above in MATH 20, or 2 yrs college-prep algebra + suitable math placement score, or concurrent enrollment in MATH 50 + suitable placement score, or grade of C or above in MATH 50. - May not be counted for credit toward a major or minor in mathematics with the exception of the Middle Childhood/Early Adolescence minor in mathematics or the Mathematical Problem Solving minor.

Basic statistical analysis, including descriptive statistics, probability, confidence intervals, hypothesis testing, simple linear regression, correlation, Chi-Square, and Analysis of Variance.

Attributes: GE IB Communication-Analytical Skills, LE-S2 Mathematics Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 291 Special Topics (1-3 crs)

Topics will vary, depending on interest of students and faculty.
Repeat: Course may be repeated for a maximum of 9 credits

## MATH 297 Developmental Tutoring Techniques (1 cr)

- May not be counted for credit toward a major or minor in mathematics except in the Comprehensive Major: Mathematics Teaching emphasis or the Minor: Mathematics, Middle Childhood/Early Adolescence, Teaching or the Minor: Mathematical Problem Solving, Liberal Arts.

Training and tutoring for developmental mathematics courses.
Repeat: Course may be repeated for a maximum of 4 credits
Seminar Hours: 1

## MATH 302 Algebraic Thinking (2 crs)

Prerequisite: MATH 202. Limited to elementary and special education majors and minors.
Mathematical knowledge for teachers based on practice/content standards. Topics include: algebraic equations, linear functions, non-linear functions, and introduction to formal justification.

Lecture/Discussion Hours: 2
Lab/Studio Hours: 0

## MATH 303 Probability and Statistical Thinking (2 crs)

Prerequisite: MATH 202. Limited to elementary and special education majors and minors.
Mathematical knowledge for teachers based on practice/content standards. Topics include: statistics, probability, and expected value.

Lecture/Discussion Hours: 2
Lab/Studio Hours: 0

## MATH $\mathbf{3 0 4}$ Geometric Thinking (2 crs)

Prerequisite: MATH 202. Limited to elementary and special education majors and minors.
Mathematical knowledge for teachers based on practice/content standards. Topics include: 2D \& 3D figures, and transformations.

Lecture/Discussion Hours: 2
Lab/Studio Hours: 0

MATH 305 Mathematics for Elementary/Middle School Teachers III (3 crs)
Prerequisite: MATH 204 or concurrent enrollment. Exceptions to the MATH 204 prerequisite may be made with the instructor's approval. Limited to elementary and special education majors and minors. Minimum junior standing.

- Counts only toward the elementary teaching minor in mathematics.

An integrated study of mathematics content, methods, and curricula. Topics include statistics, measurement, geometries, patterns, and functions. Emphasis on communication and algebraic thinking. Practicum experience included.

Attributes: GE IB Communication-Analytical Skills
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 307 Mathematics and Music (3 crs)

Prerequisite: MATH 114 (or equivalent) or consent of instructor. Using mathematics to understand, appreciate, and create music. Free audio processing software will be emphasized. Rock, jazz, classical, and experimental music will be analyzed. Connections are made to brain science and medicine.

Attributes: GE V University Wide, LE-I1 Integration, LE-S3 Creativity, Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

MATH 311 Differential Equations (3 crs)
Prerequisite: MATH 216. Credit may not be earned in both MATH 311 and MATH 312.
Ordinary differential equations of first and second order, linear differential equations, Laplace transforms, series solutions, and systems of linear differential equations.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 312 Differential Equations and Linear Algebra (4 crs)

Prerequisite: MATH 215. Credit may not be earned in both MATH 311 and MATH 312.
Linear algebra: basis, dimension, matrix algebra, determinants, inverses, systems of linear equations, eigenvalues/eigenvectors. (Optional) matrices as linear transformations. Differential equations: first-order linear, separable; second-order linear with constant coefficients; higher order differential equations; first-order linear systems with constant coefficients; Laplace transforms; power series solutions. (Optional) Proof of Existence and Uniqueness Theorems.

Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 313 Digital Signal Processing ( 3 crs )

Prerequisite: MATH 215 or consent of instructor.
Fundamentals of digital signal processing. Fourier series, Fourier transforms and computerized Fourier transforms (FFTs) are described and applied to the analysis of digitized audio and digitized images.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 314 Discrete Mathematics (3 crs)

Prerequisite: MATH 114 or MATH 212. No credit if taken after CS 240. Introductory course in discrete mathematics. Topics include elementary combinatorics, graph theory, trees, and Boolean algebra. The course emphasizes an algorithmic approach to problem solving.

Attributes: GE IB Communication-Analytical Skills, Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 315 Advanced Calculus I (3 crs)

Prerequisite: MATH 216 and MATH 324.
Differential and integral calculus of several variables; vector calculus; line, surface, and space integrals including Green's theorem, Divergence theorem, and Stokes' theorem.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 316 Introduction to Real Analysis (3 crs)

Prerequisite: MATH 216 and MATH 324.
Real numbers, introductory topological topics, limits, sequences of numbers, continuity, sequences of functions, and series.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 317 Introduction to Real Analysis II (3 crs)

Prerequisite: MATH 316
Differentiation and integration in Rn, Riemann-Stieltjes integral, functions of bounded variation, implicit function theorem, introduction to metric spaces including Arzela's Theorem and Tietze extension Theorem, and selected topics in analysis.

Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 318 Introduction to Complex Variables (3 crs)

Prerequisite: MATH 216
Elementary functions of a complex variable, complex integrals, and residue theory.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 322 Abstract Algebra for Elementary Teachers (4 crs)

Prerequisite: MATH 114 and MATH 202. Limited to elementary education majors.

- May not be counted for credit toward a major or minor in mathematics except in mathematics teaching programs or in the minor in Mathematical ProblemSolving.

Includes sets, functions, and abstract algebraic structures such as groups, rings, and fields.

Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 324 Linear Algebra (4 crs)

Prerequisite: MATH 215
An introductory course in linear algebra including matrix algebra, systems of linear equations, vector spaces, linear transformations, eigenvalues, and applications. Also covers methods of proof and a survey of student-faculty research.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 330 Modern Geometry (4 crs)

Prerequisite: MATH 114

- Requires basic knowledge of Euclidean Geometry.

Contemporary and classical Euclidean geometry, with emphasis on constructions, transformations, and proofs. The Mira and geometry software will be used to illustrate these topics. Some non-Euclidean geometry included.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 335 Introduction to Point-Set Topology (3 crs)

Prerequisite: MATH 324
Theory of sets, continuity of functions, neighborhoods of points, topological and metric spaces, connectedness and compactness.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 338 Introduction to Differential Geometry (3 crs)

Prerequisite: MATH 216
An introduction to the study of curves and surfaces. Topics on curves include curvature, torsion, and the Frenet frames. Topics on surfaces include curvature, minimal surfaces, and Gauss' Remarkable Theorem.

Lecture/Discussion Hours: 2.5
Lab/Studio Hours: 0.5

## MATH 339 Relativity and Differential Geometry (4 crs)

Prerequisite: MATH 216, PHYS 232
The theories and history of special and general relativity. Manifolds, tensors, and metrics as they relate to relativity. Modern research in relativity and astronomy.

Attributes: LE-I1 Integration
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 341 Classical Number Theory (3 crs)

Prerequisite: MATH 114
Classical number theory including divisibility, primes and their properties, theory of congruences, Diophantine equations, number theoretic functions, primitive roots, and selected optional topics.

Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

MATH 345 Introduction to Probability and Mathematical Statistics (4 crs) Prerequisite: MATH 215 or concurrent registration. Credit may not be earned in both MATH 345 and MATH 346.

- MATH 246 may not be taken for credit after or concurrently with MATH 345.

Counting techniques, discrete and continuous random variables, probability distributions, sampling distributions, estimation, hypothesis testing, linear regression, correlation, nonparametric statistics. Students who desire more extensive probability and statistics should take Math 346/Math 546, Math 347/ Math 547.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 346 Introduction to Probability (4 crs)

Prerequisite: MATH 216 or concurrent enrollment. Credit may not be earned in both MATH 345 and MATH 346.
Probability in discrete and continuous sample spaces; conditional probability; counting techniques; probability functions; binomial, Poisson, normal distributions; and transformations of variables.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 347 Mathematical Statistics (4 crs)

Prerequisite: MATH 215; MATH 346; DS 140.

- MATH 246 may not be taken for credit after or concurrently with MATH 347.

Sampling distributions; estimation and properties; hypothesis testing; analysis of variance; regression; nonparametric tests; and introduction to Bayesian inference.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 2

## MATH 350 Introduction to Financial Mathematics (4 crs)

Prerequisite: MATH 215 or consent of instructor.
Mathematical theory of interest, annuities and applications, loans, portfolio management, securities valuation, and an introduction to financial derivatives.

Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 351 Numerical Analysis I (3 crs)

Prerequisite: MATH 215
Nonlinear equations, interpolation and approximation, least squares, systems of linear equations, and error analysis.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

MATH 354 Introduction to Mathematical Modeling (4 crs)
Prerequisite: MATH 215
Introduction to building mathematical models in an applied context, including principles of modeling; project(s) involve modeling open-ended real-world problems. Skills covered may include discrete dynamical systems, differential equations, stochastic models, and linear programming.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 365 Patterns of Problem Solving ( $\mathbf{4} \mathbf{~ c r s ) ~}$

Prerequisite: Minimum-level University Mathematics Competency. - May not be counted for credit toward a major or minor in mathematics except in mathematics teaching programs or in the minor in Mathematical ProblemSolving.

Provides tools and concepts needed in problem solving in a variety of disciplines.

Attributes: GE IB Communication-Analytical Skills, LE-S3 Creativity Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 373 Issues in Secondary Mathematics (4 crs)

Prerequisite: MATH 345 or MATH 346 or concurrent enrollment; MATH 330 or concurrent enrollment. Limited to mathematics teaching majors.

- This is a hybrid course offering.

An integrated study of mathematical topics taught in grades 5-12 with a focus on increasing pedagogical content knowledge. Topics covered include algebraic, geometric, and other relevant issues in secondary mathematics.

Grading Basis: A-F Grades Only
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 380 Research Methods (3 crs)

Prerequisite: MATH 324 or consent of instructor.
Introduction to research methods in mathematics. Topics include: background research, methods of proof, mathematical writing, and presentation skills.

Attributes: LE-S3 Creativity
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

MATH 395 Directed Studies (1-3 crs)
Prerequisite: 2.00 GPA in mathematics.
Consent: Department Consent Required
For the student who wishes to pursue independent study of a topic in mathematics under the supervision of a member of the mathematics department.

Repeat: Course may be repeated

MATH 399 Independent Study - Juniors (1-3 crs)
Prerequisite: Minimum junior standing.
Consent: Department Consent Required
Individual project under the direction of a faculty member.
Repeat: Course may be repeated

## MATH 425 Abstract Algebra I (3 crs)

Prerequisite: MATH 324
An introduction to the abstract concepts of algebra such as groups, rings, and fields. Topics such as direct products, group homomorphisms, factor groups, integral domains, and polynomial rings will be included.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 426 Abstract Algebra II (3 crs)

Prerequisite: MATH 425/MATH 625. No credit if taken after MATH 322.
A continuation of Math 425 including more advanced topics from groups, rings, and fields, such as the Sylow theorems, prime and maximal ideals, unique factorization domains, and finite fields.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 440 Digital Image Processing ( $\mathbf{3}$ crs)

Prerequisite: MATH 215

- Credit may not be earned in both MATH 440 and PHYS 440.

Applications of Fourier analysis and wavelets to optics and image processing. Topics include: diffraction, wave optical theory of lenses and imaging, wavelets, and image processing.

Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 441 Linear Regression Analysis, with Time Series (4 crs)

Prerequisite: MATH 345 or MATH 347 or MATH 246, and MATH 114, and DS 140.
Simple and multiple regression, transformation of variables, model checking, analysis of residuals, regression diagnostics, stepwise regression, index variables, confidence intervals, and computer software; overview of time series and forecasting methods.

Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 442 Advanced Statistical Modeling (4 crs)

Prerequisite: MATH 345 or MATH 347 or MATH 246, and DS 140.
Teaches students the key concepts of generalized linear models, principal components analysis, decision tree models, and cluster analysis. Students will learn the purpose and uses of these methods, make interpretation of obtained results, and gain knowledge of their various applications. Applications will be implemented using the statistical software R / Rstudio.

Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 443 Experimental Design and Analysis (3 crs)

Prerequisite: MATH 345 or MATH 347 or MATH 246, and DS 140.
Basic concepts of statistical design of experiments; linear models; randomized block, Latin square, factorial, fractional factorial, rotatable, and incomplete block designs and applications.

Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 445 Survey Sampling (2 crs)

Prerequisite: MATH 345 or MATH 347 or MATH 246, and DS 140.
Survey sampling including random sampling, stratified sampling, cluster sampling, and design of surveys.

Lecture/Discussion Hours: 2
Lab/Studio Hours: 0

## MATH 447 Nonparametric Statistics (2 crs)

Prerequisite: MATH 345 or MATH 347 or MATH 246, and DS 140.
Testing and estimation based on the binomial distribution, contingency tables, Wilcoxon Tests, rank correlation, Kruskal-Wallis and Friedman analysis of variance. Comparisons with parametric methods.

Lecture/Discussion Hours: 2
Lab/Studio Hours: 0

## MATH 450 Foundations of Actuarial Science (4 crs)

Prerequisite: Admission to the Actuarial Science program
Actuarial theory of financial mathematical models for contingent events as applied to insurance, annuities, pensions, and reserving. Together with Math 460 , this course prepares students for the actuarial exam covering contingent payments.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 451 Teaching Mathematics with Technology (4 crs)

Prerequisite: ES 203 or ES 210/ ES 211 or ES 212; MATH 114. Credit may not be earned in both MATH 451/MATH 651 and MATH 453/MATH 653.

- May not be counted for credit toward a major or minor in mathematics except in mathematics teaching programs or in the minor in Mathematical ProblemSolving.

In this course students will investigate teaching mathematics effectively with technology. Technology used includes graphing calculators, spreadsheets, interactive software, and calculator based laboratories. Topics include functions, statistics, probability, geometry, and mathematical modeling.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 460 Contingent Payment Analysis (4 crs)

Prerequisite: MATH 450 and Admission to the Actuarial Science Program (Code 180-001).
Actuarial theory of reserves, multiple life models, multiple decrement models, Poisson processes, and multi-state transition models. Together with Math 450, this course prepares students for the actuarial exam covering contingent payments.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 462 History of Mathematics (3 crs)

Prerequisite: MATH 114
A study of mathematics from a historical point of view beginning with the work of the early Greek mathematicians and including selected mathematicians and periods of mathematical development up to the present.

Attributes: LE-I1 Integration, Undergraduate/Graduate Offering
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 470 Mathematical Models for Financial Economics (4 crs)

Prerequisite: Admission to the Actuarial Science Program (Code 180-001).
An introduction to financial derivatives including forwards and options with coverage of binomial, Black-Scholes, and lognormal pricing models.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 475 Actuarial Foundations of Short-Term Coverages (4 crs)

Prerequisite: MATH 347. Admitted to Actuarial Science Program. No credit if taken after MATH 491 when offered as Introduction to Credibility \& Loss Models An introduction to ratemaking and reserving for short-term insurance coverages; credibility theory; and actuarial models for frequency, severity, and aggregate distributions.

Attributes: Undergraduate/Graduate Offering
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 480 Research Seminar (2 crs)

Prerequisite: MATH 380 and faculty supervised research project
Consent: Instructor Consent Required
Individual research projects in mathematics under the supervision of a member of the mathematics faculty, culminating in research project presentations.

Repeat: Course may be repeated for a maximum of 4 credits
Lab/Studio Hours: 0
Seminar Hours: 2

## MATH 491 Special Topics (1-4 crs)

A variable content course designed to allow a breadth of study through investigation of mathematical topics not covered in other courses. Special interests of instructors will be utilized to provide topics.

Attributes: Undergraduate/Graduate Offering
Repeat: Course may be repeated for a maximum of 9 credits

## MATH 493 Capstone Seminar (1 cr)

Prerequisite: Minimum senior standing.
Consent: Department Consent Required
An intensive study of selected topics including written and/or oral
presentations by the students. The topics will connect ideas from across the mathematics major.

Attributes: Capstone Course
Repeat: Course may be repeated for a maximum of 3 credits
Lecture/Discussion Hours: 1
Lab/Studio Hours: 0

## MATH 494 Mathematics Seminar (1 cr)

Prerequisite: Minimum junior standing.
An intensive study of selected topics in mathematics. The exact topics to be studied will vary according to the interests of the professor and the seminar participants.

Attributes: Undergraduate/Graduate Offering
Repeat: Course may be repeated for a maximum of 3 credits
Lecture/Discussion Hours: 1
Lab/Studio Hours: 0

## MATH 497 Honors Seminar (1 cr)

Prerequisite: Grade of B+ or above in MATH 216.

- Admission to Department Honors Program.

An intensive study of selected topics in mathematics. The exact topics to be studied will vary according to the interests of the professor and the seminar participants.

Repeat: Course may be repeated for a maximum of 3 credits
Lecture/Discussion Hours: 1
Lab/Studio Hours: 0

## MATH 498 Mathematics Internship (1-3 crs)

Prerequisite: MATH 216. Limited to mathematics majors and minors. Minimum junior standing.
Consent: Department Consent Required - May not count toward the minimal number of credits required for a mathematics major or minor.

Provides supervised on-the-job experience in an area of interest with mathematical content.

Attributes: Service-Learning Optional, Internship
Repeat: Course may be repeated for a maximum of 6 credits Grading Basis: No Auditors

## MATH 499 Independent Study - Seniors (1-3 crs)

Prerequisite: Minimum senior standing.
Consent: Department Consent Required
Individual project under the direction of a faculty member.
Repeat: Course may be repeated

## MATH 507 Mathematics and Music ( 3 crs)

Prerequisite: MATH 114 (or equivalent) or consent of instructor.

- Dual-listed with MATH 307. Credit may not be earned in both courses.

Using mathematics to understand, appreciate, and create music. Free audio processing software will be emphasized. Rock, jazz, classical, and experimental music will be analyzed. Connections are made to brain science and medicine.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 511 Differential Equations (3 crs)

Prerequisite: MATH 216. Credit may not be earned in both MATH 311/ MATH 511 and MATH 312.

- Cross-listed with MATH 311. Credit may not be earned in both courses.

Ordinary differential equations of first and second order, linear differential equations, Laplace transforms, series solutions, and systems of linear differential equations.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 513 Digital Signal Processing (3 crs)

Prerequisite: MATH 215 or consent of instructor.

- Dual-listed with MATH 313. Credit may not be earned in both courses.

Fundamentals of digital signal processing. Fourier series, Fourier transforms and computerized Fourier transforms (FFTs) are described and applied to the analysis of digitized audio and digitized images.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 514 Discrete Mathematics (3 crs)

Prerequisite: MATH 114 or MATH 212. No credit if taken after CS 240.

- Dual-listed with MATH 314. Credit may not be earned in both courses.

Introductory course in discrete mathematics. Topics include elementary combinatorics, graph theory, trees, and Boolean algebra. The course emphasizes an algorithmic approach to problem solving.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 515 Advanced Calculus I (3 crs)

Prerequisite: MATH 216 and MATH 324/MATH 524.

- Dual-listed with MATH 315. Credit may not be earned in both courses.

Differential and integral calculus of several variables; vector calculus; line, surface, and space integrals including Green's theorem, Divergence theorem, and Stokes' theorem.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 516 Introduction to Real Analysis (3 crs)

Prerequisite: MATH 216 and MATH 324/MATH 524.

- Dual-listed with MATH 316. Credit may not be earned in both courses.

Real numbers, introductory topological topics, limits, sequences of numbers, continuity, sequences of functions, and series.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 518 Introduction to Complex Variables (3 crs)

## Prerequisite: MATH 216

- Dual-listed with MATH 318. Credit may not be earned in both courses.

Elementary functions of a complex variable, complex integrals, and residue theory.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 524 Linear Algebra (4 crs)

Prerequisite: MATH 215

- Dual-listed with MATH 324. Credit may not be earned in both courses.

An introductory course in linear algebra including matrix algebra, systems of linear equations, vector spaces, linear transformations, eigenvalues, and applications. Also covers methods of proof and a survey of student-faculty research.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 530 Modern Geometry (4 crs)

Prerequisite: MATH 114

- Dual-listed with MATH 330. Credit may not be earned in both courses. Requires basic knowledge of Euclidean Geometry.

Contemporary and classical Euclidean geometry, with emphasis on constructions, transformations, and proof. The Mira and geometry software will be used to illustrate these topics. Some non-Euclidean geometry included.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 535 Introduction to Point-Set Topology (3 crs)

Prerequisite: MATH 324/MATH 524

- Dual-listed with MATH 335. Credit may not be earned in both courses.

Theory of sets, continuity of functions, neighborhoods of points, topological and metric spaces, connectedness and compactness.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

MATH 545 Introduction to Probability and Mathematical Statistics (4 crs)
Prerequisite: MATH 215 or concurrent registration. Credit may not be earned in both MATH 345/MATH 545 and MATH 346/MATH 546.

> - Dual-listed with MATH 345. Credit may not be earned in both courses. MATH 246 may not be taken for credit after or concurrently with MATH 345/ MATH 545.

Counting techniques, discrete and continuous random variables, probability distributions, sampling distributions, estimation, hypothesis testing, linear regression, correlation, nonparametric statistics. Students who desire more extensive probability and statistics should take MATH 346/MATH 546, MATH 347/MATH 547.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 546 Introduction to Probability (4 crs)

Prerequisite: MATH 216 or concurrent enrollment. Credit may not be earned in both MATH 345/MATH 545 and MATH 346/MATH 546.

- Dual-listed with MATH 346. Credit may not be earned in both courses.

Probability in discrete and continuous sample spaces; conditional probability; counting techniques; probability functions; binomial, Poisson, normal distributions; and transformations of variables.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 547 Mathematical Statistics (4 crs)

Prerequisite: MATH 215; MATH 346/MATH 546; DS 140.

- Dual-listed with MATH 347. Credit may not be earned in both courses.

MATH 246 may not be taken for credit after or concurrently with MATH 347/ MATH 547.

Sampling distributions; estimation and properties; hypothesis testing; analysis of variance; regression; nonparametric tests; and introduction to Bayesian inference.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 2

## MATH 551 Numerical Analysis I (3 crs)

Prerequisite: MATH 215

- Dual-listed with MATH 351. Credit may not be earned in both courses.

Nonlinear equations, interpolation and approximation, least squares, systems of linear equations, and error analysis.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 554 Introduction to Mathematical Modeling (4 crs)

## Prerequisite: MATH 215

- Dual-listed with MATH 354. Credit may not be earned in both courses.

Introduction to building mathematical models in an applied context, including principles of modeling; project(s) involve modeling open-ended real-world problems. Skills covered may include discrete dynamical systems, differential equations, stochastic models, and linear programming.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 625 Abstract Algebra I (3 crs)

Prerequisite: MATH 324/MATH 524

- Dual-listed with MATH 425. Credit may not be earned in both courses.

An introduction to the abstract concepts of algebra such as groups, rings, and fields. Topics such as direct products, group homomorphisms, factor groups, integral domains, and polynomial rings will be included.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 626 Abstract Algebra II (3 crs)

Prerequisite: MATH 425/MATH 625. No credit if taken after MATH 322.

- Dual-listed with MATH 426. Credit may not be earned in both courses.

A continuation of Math 425 including more advanced topics from groups, rings, and fields, such as the Sylow theorems, prime and maximal ideals, unique factorization domains, and finite fields.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 650 Foundations of Actuarial Science (4 crs)

Prerequisite: Admission to Actuarial Science program (Code 180-001).

- Dual-listed with MATH 450. Credit may not be earned in both courses.

Actuarial theory of financial mathematical models for contingent events as applied to insurance, annuities, pensions, and reserving. Together with Math 460, this course prepares students for the actuarial exam covering contingent payments.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 651 Teaching Mathematics with Technology (4 crs)

Prerequisite: ES 203 or ES 210/ES 211 or ES 212; MATH 414. Credit may not be earned in both MATH 451/MATH 651 and MATH 453/MATH 653.

- Dual-listed with MATH 451. Credit may not be earned in both courses. May not be counted for credit toward a major or minor in mathematics except in mathematics teaching programs or in the minor in Mathematical ProblemSolving.

In this course students will investigate teaching mathematics effectively with technology. Technology used includes graphing calculators, spreadsheets, interactive software, and calculator based laboratories. Topics include functions, statistics, probability, geometry, and mathematical modeling.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 660 Continent Payment Analysis (4 crs)

Prerequisite: MATH 450/MATH 650 and Admission to the Actuarial Science Program (Code 180-001).

- Dual-listed with MATH 460. Credit may not be earned in both courses.

Actuarial theory of reserves, multiple life models, multiple decrement models. Poisson processes, and multi-state transition models. Together with Math 450 , this course prepares students for the actuarial exam covering contingent payments.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 662 History of Mathematics ( $\mathbf{3}$ crs)

Prerequisite: MATH 114

- Dual-listed with MATH 462. Credit may not be earned in both courses.

A study of mathematics from a historical point of view beginning with the work of the early Greek mathematicians and including selected mathematicians and periods of mathematical development up to the present.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 670 Mathematical Models for Financial Economics (4 crs)

Prerequisite: Admission to the Actuarial Science Program (Code 180-001).

- Dual-listed with MATH 470. Credit may not be earned in both courses.

An introduction to financial derivatives including forwards and options with coverage of binomial, Black-Scholes, and lognormal pricing models.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 675 Actuarial Foundations of Short-Term Coverages (4 crs)

Prerequisite: MATH 347. Admission to Actuarial Science program. No credit if
taken after MATH 491 when offered as Introduction to Credibility \& Loss Models

- Dual-listed with MATH 475. Credit may not be earned in both courses.

An introduction to ratemaking and reserving for short-term insurance coverages; credibility theory; and actuarial models for frequency, severity, and aggregate distributions.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 4
Lab/Studio Hours: 0

## MATH 691 Special Topics (1-4 crs)

- Dual-listed with MATH 491.

A variable content course designed to allow a breadth of study through investigation of mathematical topics not covered in other courses. Special interests of instructors will be utilized to provide topics.

Repeat: Course may be repeated for a maximum of 9 credits
Grading Basis: No S/U Grade Option

## MATH 694 Mathematics Seminar (1 cr)

- Dual-listed with MATH 494.

An intensive study of selected topics in mathematics. The exact topics to be studied will vary according to the interests of the professor and the seminar participants.

Repeat: Course may be repeated for a maximum of 3 credits
Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 1
Lab/Studio Hours: 0

## MATH $\mathbf{7 0 4}$ Mathematics for Middle School Teachers (3 crs)

Topics include number and number sense, probability, statistics, reasoning, problem solving, patterns and functions, geometry, measurement, and algebra.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 707 Current Issues in Mathematics Teaching (3 crs)

A particular current trend in mathematics education will be the theme for each offering. Focus will be on mathematical content necessary to design instruction and develop curriculum. Nonthematic issues will also be discussed.

Repeat: Course may be repeated
Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 710 Selected Topics in Calculus (1-3 crs)

Prerequisite: Minimum eight credits of Calculus or consent of instructor Consent: Instructor Consent Required
An in-depth investigation of elementary calculus from an advanced viewpoint, including functions, limits, continuity, integrals, derivatives, and related applications.

Grading Basis: No S/U Grade Option

## MATH 719 Introduction to Real Analysis (3 crs)

Prerequisite: MATH 215
Selected topics including sets, the real number system, mappings, sequences, limits, continuity, the derivative, and the integral.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 721 Foundations of Algebra (3 crs)

Prerequisite: MATH 425/MATH 625
Set theory, mappings, the integers, groups, rings, and vector spaces.
Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 731 Vector Methods in Geometry ( $\mathbf{3} \mathbf{c r s}$ )

Applications of vector methods to Euclidean geometry.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 732 Advanced Geometry (3 crs)

Prerequisite: One course in Calculus
Advanced study of geometry including Euclidean, non-Euclidean, projective, and affine geometries.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 742 Logic and Mathematics ( 3 crs )

Symbolic logic and its use in mathematics, including proof, demonstration, and applications in abstract mathematical systems.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

MATH 748 Applied Statistical Procedures ( $\mathbf{3} \mathbf{~ c r s )}$
Prerequisite: MATH 246 or consent of instructor.
Includes methods of estimation, parametric and non-parametric tests of significance, linear regression and correlation, and analysis of variance for one-way or two-way crossed classified data. Applications include the use of computers.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 749 Probability and Statistics Workshop (3 crs)

Consent: Instructor Consent Required
Applications of probability and statistics which can be used by the secondary school teacher to motivate and enrich traditional mathematics classes and as topics in upper-level classes where some course content in probability or statistics is desired.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 751 Technology in Mathematics (3 crs)

Prerequisite: MATH 216

- Requires basic knowledge of Euclidean geometry.

The use of graphing calculators and computer software to enhance understanding of mathematical concepts and to formulate statements of theorems and solutions to problems is explored.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 755 Linear and Nonlinear Programming ( 3 crs )

Prerequisite: MATH 216 and MATH 324/MATH 524.
Introductory topics in linear and nonlinear programming including the simplex method for solving a linear program and Wolf's method for solving a quadratic program; and duality theory for linear, quadratic, and nonlinear programming.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 765 Number Theory ( 3 crs)

Foundations of number theory emphasizing congruences, selected number theoretic functions, Diophantine equations, continued fractions, and other classical problems.

Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 791 Special Topics (3 crs)

Prerequisite: Limited to MAT/MST MATH or MEPD program.

- Permission of academic program adviser required.

A variable content course designed to allow breadth of study through investigation of mathematical topics not covered in other courses.

Repeat: Course may be repeated for a maximum of 12 credits
Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 3
Lab/Studio Hours: 0

## MATH 793 Directed Studies (1-3 crs)

Permits students to undertake independent studies under the direction of a graduate faculty member.

Repeat: Course may be repeated
Grading Basis: No S/U Grade Option

MATH 794 Graduate Mathematics Seminar (1 cr)
Consent: Instructor Consent Required Lab/Studio Hours: 0

Repeat: Course may be repeated
Grading Basis: No S/U Grade Option
Lecture/Discussion Hours: 1

MATH 795 Research Paper (1-2 crs)
Consent: Instructor Consent Required
Repeat: Course may be repeated
Grading Basis: PR Only Grade Basis

MATH 797 Independent Study (1-3 crs)
Consent: Department Consent Required
Individual project under the direction of a faculty member.
Repeat: Course may be repeated Grading Basis: No S/U Grade Option

## MATH 799 Thesis (1-6 crs)

Consent: Instructor Consent Required

- Full-time equivalent.

A description of acceptable topics and the precise nature of the thesis requirement is provided in the departmental program descriptions.

Repeat: Course may be repeated
Grading Basis: PR Only Grade Basis

